ST. CATHERINE'S SCHOOL

YEAR 12 TRIAL EXAMINATION

3/4 UNIT MATHEMATICS

TIME ALLOWED: 2 HOURS (*PLUS 5 MINUTES READING TIME*)

DATE: AUGUST 1999

STUDENT NUMBER:______________________________

DIRECTIONS TO CANDIDATES:

- This paper consists of seven questions.
- All questions are to be attempted.
- All questions are of equal value.
- In every question, all necessary working should be shown.
- Marks may be deducted for careless or badly arranged work.
- Approved calculators and geometrical instruments are required.
- Begin a NEW PAGE for every question.
- Attach your question paper to the front of Section A.
- Hand your work in three bundles:

  Section A - Questions 1, 2 and 3

  Section B - Questions 4, 5, 6 and 7

- This sheet will form the cover page for Section A. You will need to write a cover sheet for Section B, which clearly states your Student Number.

Securely staple or tie questions together in sections.

TEACHERS USE ONLY TOTAL MARKS
A
B

3 Unit Trial Mathematics Examination Paper 1999
Section A
Question 1

Marks

a) Solve the inequality $\frac{x^2-1}{x} > 0$ 3

b) Evaluate $\int_0^{\pi} \sin^2 x\, dx$ 3

c) Integrate $\int \frac{t}{\sqrt{1+t}}\, dt$ by using the substitution $t = u^2 - 1$ 3

d) A particle moves from rest from the origin in a straight line in such a way that its velocity $v\ m/s$ is given by $v = 20t - 5t^2$, (where t is in seconds). 3

Find (i) when the particle comes to rest

(ii) the greatest velocity of the particle.

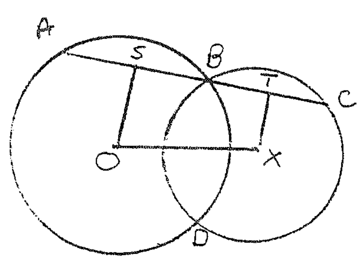
Question 2 (Start a new page)

a) If $A(x)$ is a factor of $P(x)$, find a when $A(x) = x - 4$ and $P(x) = x^3 + 2x^2 + ax - 20$ 2

b) Express $12\cos\theta + 5\sin\theta$ in the form $R\cos(\theta - \alpha)$ and use it to solve $12\cos\theta + 5\sin\theta = 13$ for $0^0 \le \theta \le 360^0$. 5

c) The equation $e^x = x + 2$ has a root close to $x = 1.2$. Use Newton's method once to find a better approximation to this root (correct to 2 decimal places). 2

d) ABC is a straight line 3
S and T are midpoints of AB and BC respectively
O is centre of cirlce ABD
X is centre of circle BCD



Prove <SOX is the supplement of <OXT.

Question 3 (Start a new page)

Marks

a) Consider the function $f(x) = \frac{x}{x^2+1}$ 7

(i) Show that it is an odd function

(ii) Find any stationary points and given that $f''(x) = \frac{2x(x^2-3)}{(x^2+1)^3}$, find any points of inflexion.

(iii) Describe the behaviour of $f(x)$ for very large positive and very large negative values of x i.e. when $x \to \infty$ and $x \to -\infty$.

(iv) Sketch the curve.

b) Prove by mathematical induction that
$\sum_{r=1}^{n} r(r+2) = \frac{1}{6}n(n+1)(2n+7)$ where n is a positive integer. 5

SECTION B (Start a new page)

Question 4

Marks

a) (i) How many odd 4 digit numbers can be made from the digits 2, 3, 4, 5, 6 if none of the digits are repeated? 3

(ii) What is the probability of an odd number being selected if the digits may be repeated?

b) Without using your calculator, evaluate 3

(i) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

(i) $\cos\left(\sin^{-1}\left(-\frac{3}{5}\right)\right)$

c) Using the t-results, show that $\dfrac{\cot\frac{\theta}{2}+\tan\frac{\theta}{2}}{\cot\frac{\theta}{2}-\tan\frac{\theta}{2}}=\sec\theta$ 3

d) Evaluate $\int_0^{\frac{1}{4}}\frac{dx}{1+16x^2}$ 3

Question 5 (Start a new page)

a) The sum of three acute angles is 45^0 and the tangent ratios of two of them are $\frac{1}{2}$ and $\frac{1}{4}$ respectively. Without using your calculator, find the tangent ratio of the third angle. 4

b) The normals to the parabola $x^2 = 4ay$ at the points $P\left(2ap, ap^2\right)$ and $Q\left(2aq, aq^2\right)$ intersect at R. 8

(i) Derive the equation of the normal at P

(ii) Find R, the point of intersection of the normals at P and Q

(iii) Derive the equation of the chord PQ.

(iv) If the chord PQ varies in such a way that it always passes through $(0,2a)$ find the locus of R.

Question 6 (Start a new page)

Marks

a) Find the co-ordinates of the point P which divides the interval AB with end points $A(2,3)$ and $B(5,-7)$ internally in the ratio 4:9. **2**

b) A sphere is expanding such that its surface area is increasing at the rate of $0.01\,cm/\sec^2$. Calculate the rate of change of **5**

(i) its radius
(ii) its volume

at an instant when the radius is 5 cm.

c) Find $\frac{d}{dx}\sin^{-1}e^{2x}$ and hence evaluate $\int_{-\ln\sqrt{2}}^{0} \frac{2e^{2x}}{\sqrt{1-e^{4x}}}\,dx$ **5**

Question 7 (Start a new page)

Marks

a) Brine, containing 1 kg of salt per 10 litres, runs into a tank, initially filled with 500 litres of fresh water, at a rate of 25 litres per minute. 6

The mixture runs out of the tank at the same rate of 25L/min.

(i) If A is the amount of salt in the tank at time t, by calculating the concentration of salt flowing in and out of the tank,

show that $\frac{dA}{dt} = -\frac{1}{20}(A-50)$

NOTE: 1 L of water weighs 1kg.

(ii) Find the amount of salt in the tank at the end of 100 minutes, assuming that the mixture is kept uniform by stirring.

b) A particle moves with an acceleration which varies linearly as the distance travelled such that $\ddot{x} = mx + b$. It starts at the origin from rest with an accleleration of $3m/s^2$ and reaches maximum speed in a distance of $160m$. 6

Find (i) the maximum speed

(ii) the speed when the particle has moved $80m$.

END OF EXAMINATION

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\quad n\neq -1;\quad x\neq 0,\ \text{if } n<0$$

$$\int \frac{1}{x}\,dx = \ln x,\quad x>0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\quad a\neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\quad a\neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\quad a\neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\quad a\neq 0$$

$$\int \sec ax\tan ax\,dx = \frac{1}{a}\sec ax,\quad a\neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\quad a\neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\quad a>0,\quad -a<x<a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\quad x>a>0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x,\quad x>0$

St Catherine's Trial HSC Solutions 1999

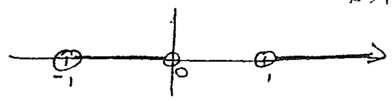
1 a) $\frac{x^2-1}{x} > 0$

Consider $\frac{x^2-1}{x} = 0$

$x \neq 0$ $\quad x^2 - 1 = 0$

$x = \pm 1$

$x > 1$ or $-1 < x < 0$ (3)



b) $\cos 2x = \cos^2 x - \sin^2 x$

$= 1 - 2\sin^2 x$

$\therefore \sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$

$\int_0^{\pi} \sin^2 x \, dx$

$= \int_0^{\pi} \frac{1}{2} - \frac{1}{2}\cos 2x \, dx$

$= \left[\frac{1}{2}x - \frac{1}{4}\sin 2x\right]_0^{\pi}$

$= \frac{\pi}{2} - \frac{1}{4}\sin 2\pi - 0 + \frac{1}{4}\sin 0$

$= \frac{\pi}{2} - \frac{1}{4} \times 0$

$= \frac{\pi}{2}$ (3)

) $\int \frac{t}{\sqrt{1+t}} \, dt$ where $t = u^2 - 1$

$\frac{dt}{du} = 2u$

$dt = 2u \, du$

$\therefore \int \frac{u^2-1}{\sqrt{1+u^2-1}} \times 2u \, du$

$= \int \frac{u^2-1}{\cancel{u}} \times 2\cancel{u} \, du$

$= 2\left[\frac{u^3}{3} - u\right] + c$ $\quad u^2 = t+1$, $u = \sqrt{t+1}$ (3)

$= \frac{2(t+1)^{3/2}}{3} - 2(t+1)^{1/2} + c$

$t = 0$
$v = 0$
$x = 0$

$v = 20t - 5t^2$

(i) rest when $v = 0$ (1)

$20t - 5t^2 = 0$

$5t(4 - t) = 0$

$t = 0 \quad t = 4$ rest after 4 sec

(ii) greatest velocity (2)

when $\frac{dv}{dt} = 0$ $\quad \frac{dv}{dt} = 20 - 10t$

$\frac{dv}{dt} = 0$ when $20 - 10t = 0$

$t = 2$

2a) $A(x) = x - 4$ is a factor of $P(x)$

$\therefore P(4) = 0$

$P(x) = x^3 + 2x^2 + ax - 20$

$P(4) = 4^3 + 2(4)^2 + 4a - 20 = 0$ (2)

$\therefore 76 + 4a = 0$

$4a = -76$

$a = -19$

(16)

b) $12\cos\theta + 5\sin\theta$

$= 13\left(\frac{12}{13}\cos\theta + \frac{5}{13}\sin\theta\right)$

$= 13(\cos\theta\cos\alpha + \sin\theta\sin\alpha)$ (2)

$= 13\cos(\theta - \alpha)$

$= 13\cos(\theta - 22°37')$

13, 5, 12, α

$\tan\alpha = \frac{5}{12}$

$\alpha = 22°37'$

$\therefore 12\cos\theta + 5\sin\theta = 13$

$13\cos(\theta - 22°37') = 13$

$\cos(\theta - 22°37') = 1$

$\therefore \theta - 22°37' = -360°, 0°, 360° \ldots$

$\theta = 22°37'$ for $0 \le \theta \le 360°$ (3)

c) $e^x = x + 2$

$\therefore e^x - x - 2 = 0$

$P(x) = e^x - x - 2$

$P'(x) = e^x - 1$

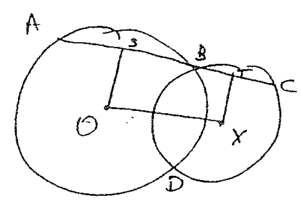
$P(1.2) = e^{1.2} - 1.2 - 2 = 0.1201$

$P'(1.2) = e^{1.2} - 1 = 2.3201$

2nd approx $x_2 = 1.2 - \frac{P(1.2)}{P'(1.2)}$ (2)

$= 1.15$

d)



S is midpt of AB data

$\angle OSA = \angle OSB = 90°$ join of centre to midpt of chord is perp to chord

Similarly

T is midpt of BC data

$\angle XTB = \angle XTC = 90°$ join of centre etc

now $\angle OSB$ & $\angle XTB$ are cointerior and add to $180°$ (3)

$\therefore OS \parallel XT$

3 a) $f(x) = \dfrac{x}{x^2+1}$

(i) $f(-x) = \dfrac{-x}{(-x)^2+1}$

$= \dfrac{-x}{x^2+1}$

$= -\left(\dfrac{x}{x^2+1}\right)$

$= -f(x)$

$\therefore$ $f(x)$ is odd fn (1)

(ii) $f'(x) = \dfrac{x^2+1\ (1) - x\ (2x)}{(x^2+1)^2}$

$= \dfrac{1-x^2}{(x^2+1)^2}$

St pt occur when $f'(x)=0$ ie $\dfrac{1-x^2}{(x^2+1)^2} = 0$

$x^2 = 1$

$x = \pm 1$

$y = \pm \frac{1}{2}$ (1)

St pts $(1, \frac{1}{2})$ & $(-1, -\frac{1}{2})$

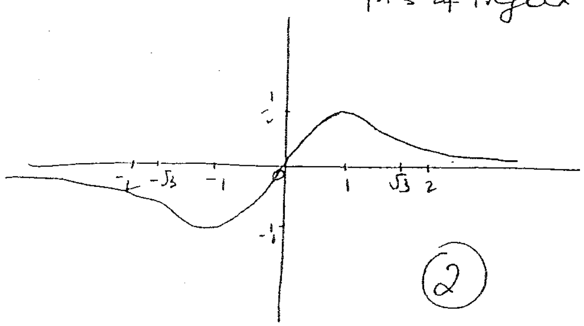
Pts of inflex may occur when $f''(x) = 0$

$\dfrac{2x(x^2-3)}{(x^2+1)^3} = 0$

ie $x = 0$ $x = \pm\sqrt{3}$

check for concavity
$x=-2$ $f''(-2) < 0$ ∩
$x=-1$ $f''(-1) > 0$ ∪
$x=1$ $f''(1) < 0$ ∩
$x=2$ $f''(2) > 0$ ∪ (2)

as there are changes of concavity pts of inflex are at $(0,0)$, $(\sqrt{3}, \frac{\sqrt{3}}{4})$, $(-\sqrt{3}, -\frac{\sqrt{3}}{4})$



(2)

(iii) as $x \to \infty$ $f(x) \to 0^+$

as $x \to -\infty$ $f(x) \to 0^-$ (1)

3b) $\displaystyle\sum_{r=1}^{n} r(r+2) = \frac{1}{6} n(n+1)(2n+7)$

1. Prove for $n=1$
LHS $= 1(1+2) = 3$ RHS $= \frac{1}{6}(1)(2)9 = 3$ (1)

2. Assume true for $n=k$
ie $\displaystyle\sum_{r=1}^{k} r(r+2) = \frac{1}{6}k(k+1)(2k+7)$ (1)

3. Prove true for $n=k+1$
ie prove $\displaystyle\sum_{r=1}^{k+1} r(r+2) = \frac{1}{6}(k+1)(k+2)(2k+9)$

Proof: LHS $= \displaystyle\sum_{r=1}^{k+1} r(r+2)$

$= \displaystyle\sum_{r=1}^{k} r(r+2) + (k+1)(k+3)$

$= \frac{1}{6}k(k+1)(2k+7) + (k+1)(k+3)$

$= (k+1)\left[\frac{1}{6}(k(2k+7) + 6(k+3))\right]$ (2)

$= \frac{1}{6}(k+1)(2k^2+7k+6k+18)$

$= \frac{1}{6}(k+1)(2k^2+13k+18)$

$= \frac{1}{6}(k+1)(k+2)(2k+9)$

$=$ RHS

4. As it is true for $n=1$
then by step 3 it is true for $n=2$
As it is true for $n=2$
then it is true for $n=3$ and so on
& therefore, by the principle of math Ind (1)
$\displaystyle\sum_{r=1}^{n} r(r+2) = \frac{1}{6} n(n+1)(2n+7)$

(must say this for $\frac{1}{2}$m)

4. a) (i) 2, 3, 4, 5, 6

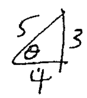
$\square\ \square\ \square\ \boxed{2} = 2 \times {}^4P_3 = 48$ (1)

ii) $\square\ \square\ \square\ \boxed{2} = 5^3 \times 2 \therefore P(\text{odd}) = \frac{5^3 \times 2}{5^4} = \frac{2}{5}$ (2)

any number $= 5^4$

) (i) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$ (1)

(ii) $\cos\left(\sin^{-1}\left(-\frac{3}{5}\right)\right)$
$= \cos(-\theta)$
$= +\frac{4}{5}$ (2)



) $\dfrac{\cot\frac{\theta}{2} + \tan\frac{\theta}{2}}{\cot\frac{\theta}{2} - \tan\frac{\theta}{2}} = \sec\theta$

let $t = \tan\frac{\theta}{2}$
$\therefore \frac{1}{t} = \cot\frac{\theta}{2}$

LHS $= \dfrac{\frac{1}{t} + t}{\frac{1}{t} - t} \begin{matrix}\times t \\ \times t\end{matrix}$

$= \dfrac{1 + t^2}{1 - t^2}$

$= \dfrac{1}{\cos\theta}$ as $\cos\theta = \dfrac{1-t^2}{1+t^2}$

$= \sec\theta$
$=$ RHS (3)

) $\displaystyle\int_0^{1/4} \frac{dx}{1+16x^2} = \int_0^{1/4} \frac{dx}{16\left(\frac{1}{16}+x^2\right)}$ (3)

$= \displaystyle\frac{1}{16}\int_0^{1/4} \frac{\frac{1}{4}}{\frac{1}{16}+x^2}\,dx$ 1

$= \dfrac{1}{16}\times 4\left[\tan^{-1}\dfrac{x}{1/4}\right]_0^{1/4}$ 1

$= \dfrac{1}{4}\left[\tan^{-1}4x\right]_0^{1/4}$

$= \dfrac{1}{4}\left[\tan^{-1}1 - \tan^{-1}0\right] = \dfrac{1}{4}\cdot\dfrac{\pi}{4} = \dfrac{\pi}{16}$ 1

5 a) $\alpha + \beta + \gamma = 45^\circ$

$\tan\alpha = \frac{1}{2}$ & $\tan\beta = \frac{1}{4}$

find $\tan\gamma$

$\tan\gamma = \tan(45 - \beta - \alpha)^\circ$
$= \tan(45 - (\alpha+\beta))^\circ$ 1

Now $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$= \dfrac{\frac{1}{2} + \frac{1}{4}}{1 - \frac{1}{2}\cdot\frac{1}{4}}$

$= \dfrac{\frac{3}{4}}{\frac{7}{8}}$

$= \frac{3}{4} \times \frac{8}{7}$ $1\frac{1}{2}$

$= \frac{6}{7}$

$\therefore \tan(45 - (\alpha+\beta))^\circ = \dfrac{\tan 45 - \tan(\alpha+\beta)}{1 + \tan 45 . \tan(\alpha+\beta)}$

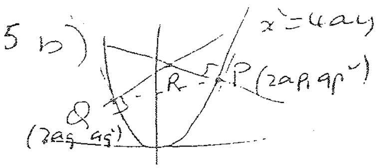
$= \dfrac{1 - \frac{6}{7}}{1 + 1\cdot\frac{6}{7}}$ $1\frac{1}{2}$

$= \dfrac{\frac{1}{7}}{\frac{13}{7}}$

$\tan(45 - (\alpha+\beta))^\circ = \frac{1}{13}$ 4

$\tan\gamma = \frac{1}{13}$

5 b)



i) $x^2 = 4ay$

$\therefore y = \frac{1}{4a}x^2$

$y' = \frac{1}{2a}x$

when $x = 2ap$ ~~y'~~ = grad of tang = $\frac{2ap}{2a}$

$= p$

$\therefore$ grad of normal $= -\frac{1}{p}$ (2)

$\therefore$ eqn of normal $y - ap^2 = -\frac{1}{p}(x - 2ap)$

$py - ap^3 = -x + 2ap$

$\therefore x + py = 2ap + ap^3$

$x + py = a(2p + p^3)$ (etc)

ii) $x + py = 2ap + ap^3$ ——— (1)

$x + qy = 2aq + aq^3$ ——— (2)

eqn (1) − (2)

$(p-q)y = 2a(p-q) + a(p^3 - q^3)$

~~$(p-q)$~~ $y = 2a$ ~~$(p-q)$~~ $+ a$ ~~$(p-q)$~~ $(p^2 + pq + q^2)$ $p \neq q$

$\therefore y = 2a + a(p^2 + pq + q^2)$ ——— (3)

Sub (3) into (1) (2)

$x + p(2a + a(p^2 + pq + q^2)) = 2ap + ap^3$

$x +$ ~~$2ap$~~ $+$ ~~ap^3~~ $+ ap^2q + apq^2 =$ ~~$2ap$~~ $+$ ~~ap^3~~

$x = -apq(p+q)$

$\therefore R(-apq(p+q), 2a + a(p^2 + pq + q^2))$

(iii) $P(2ap, ap^2)$ $Q(2aq, aq^2)$

grad of PQ $= \frac{aq^2 - ap^2}{2aq - 2ap}$

$= \frac{a(q-p)(p+q)}{2a(q-p)}$

$= \frac{p+q}{2}$ (2)

$y - ap^2 = \frac{p+q}{2}(x - 2ap)$

$y - ap^2 = \left(\frac{p+q}{2}\right)x - ap^2 - apq$

$y = \left(\frac{p+q}{2}\right)x - apq$

(iv) PQ: $y = \left(\frac{p+q}{2}\right)x - apq$

passes through $(0, 2a)$

$\therefore 2a = \left(\frac{p+q}{2}\right)0 - apq$

$pq = -2$ ——— (1)

Locus of R:

$x = -apq(p+q)$

$x = 2a(p+q)$ from (1)

$p+q = \frac{x}{2a}$

$y = 2a + a(p^2 + pq + q^2)$

$y = 2a + a((p+q)^2 - pq)$

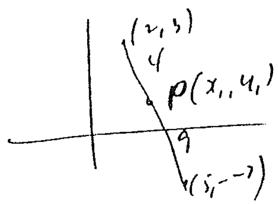
$y = 2a + a\left(\left(\frac{x}{2a}\right)^2 + 2\right)$

$y = 2a + \frac{x^2}{4a} + 2a$

$y = 4a + \frac{x^2}{4a}$

$4ay = x^2 + 16a^2$

6 a)



(2)

$x_1 = \frac{4\times 5 + 9\times 2}{4+9}$

$= 2\frac{12}{13}$

$y_1 = \frac{4\times -7 + 9\times 3}{4+9}$

$= -\frac{1}{13}$

$\therefore P\left(2\frac{12}{13}, -\frac{1}{13}\right)$

b) (i) $\frac{dS}{dt} = 0.01$ (1)

$S = 4\pi r^2$

$\frac{dS}{dt} = \frac{dS}{dr} \cdot \frac{dr}{dt}$ (1)

$\frac{dS}{dt} = 8\pi r \cdot \frac{dr}{dt}$

when $r = 5$

$0.01 = 8\pi \cdot 5 \cdot \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{0.01}{40\pi}$ (1)

$= 7.96\times10^{-5}$

rate of change of radius is

(ii) $V = \frac{4}{3}\pi r^3$ (1)

$\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$

$= 4\pi r^2 \cdot \frac{dr}{dt}$

when $r = 5$

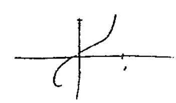
$\frac{dV}{dt} = 4\pi(25) \cdot 7.96\times10^{-5}$ (1)

$= 0.025$

6 c) $\frac{d}{dx} \sin^{-1}(e^{2x}) = \frac{1}{\sqrt{1-(e^{2x})^2}} \cdot 2e^{2x}$

$= \frac{2e^{2x}}{\sqrt{1-e^{4x}}}$ (2)

$\int_{-\ln\sqrt{2}}^{0} \frac{2e^{2x}}{\sqrt{1-e^{4x}}}\, dx = \left[\sin^{-1}(e^{2x})\right]_{-\ln\sqrt{2}}^{0}$



$= \sin^{-1}(e^{0}) - \sin^{-1}(e^{2(-\ln\sqrt{2})})$

$= \sin^{-1}(1) - \sin^{-1}(e^{-2\ln\sqrt{2}})$

$= \sin^{-1}(1) - \sin^{-1} e^{\ln(\sqrt{2})^{-2}}$

$= \frac{\pi}{2} - \sin^{-1}\frac{1}{2}$ ... $\ln(\sqrt{2})^{-2} = \frac{1}{(\sqrt{2})^2}$

$= \frac{\pi}{2} - \frac{\pi}{6}$

$= \frac{\pi}{3}$ (3)

7 a (i)

500 L, 25 L/min in, 25 L/min out

Amount of salt in tank initially is 0

$t = 0$
$A = 0$

Brine going in:

Concentration of salt $= \frac{1}{10}$ in solution

∴ Rate of salt going in $= 25 \times \frac{1}{10}$ L/min
$= \frac{25}{10}$
$= 2.5$ L/min (1)

Running out:

Concentration of salt $= \frac{A}{500} \times 25$ L/min
$= \frac{25A}{500}$
$= \frac{A}{20}$ L/min (1)

$$\frac{dA}{dt} = 2.5 - \frac{A}{20}$$
$$= \frac{50 - A}{20}$$
$$= \frac{1}{20}(50 - A)$$
$$\frac{dA}{dt} = -\frac{1}{20}(A - 50)$$ (1)

(ii) if $\frac{dA}{dt} = k(A - B)$
or $A = B + A_0 e^{kt}$ ✓ (1)

So $A = 50 + A_0 e^{-\frac{1}{20}t}$

when $T = 0$, $A = 0$ ✓

$0 = 50 + A_0 e^0$
$A_0 = -50$

∴ $A = 50 - 50e^{-\frac{1}{20}t}$

after 100 min $A = 50 - 50e^{-\frac{1}{20} \times 100}$
$= 50(1 - e^{-5})$
$= 49.66$ kg (1)

b) (i) $\ddot{x} = mx + b$

∴ $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = mx + b$ (1)

$\frac{1}{2}v^2 = \frac{m}{2}x^2 + bx + C$

$t = 0$ ✓
$x = 0$ ✓
$v = 0$ ✓
$a = 3$ ✓
max speed $x = 160$ ✓

$v^2 = mx^2 + 2bx + C_1$
$0 = 0 + C_1$
$v^2 = mx^2 + 2bx$. (1)

now $\ddot{x} = mx + b$

$a = 3$, $x = 0$: $3 = 0 + b$

$\ddot{x} = mx + 3$ (1)

It reaches max speed when $x = 160$
i.e. $\frac{dv}{dt} = 0$
∴ $\ddot{x} = 0$ when $x = 160$

$0 = 160m + 3$
$m = -\frac{3}{160}$ (1)

$v^2 = \frac{-3}{160}x^2 + 6x$

(i) max speed when $x = 160$

$v^2 = \frac{-3}{160} \cdot 160^2 + 6 \cdot 160$ (1)
$= 3 \cdot 160$
$= 480$
$v = \sqrt{480}$
$= \pm 4\sqrt{30}$ m/s

∴ max speed is $4\sqrt{30}$ m/s 21.9

(ii) speed when $x = 80$ m

$v^2 = \frac{-3}{160}(80)^2 + 6 \cdot 80$ (1)
$= 360$
$v = \pm\sqrt{360} = \pm 6\sqrt{10}$

∴ speed is $6\sqrt{10}$ m/s

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